Question

(a) Solve for x and y:

$$\frac{2x-5}{3}+\frac{y}{5}=6$$

$$\frac{3x}{10}+2=\frac{3y-5}{2}$$

(b) If $(2x-1)$ is a factor of the polynomial

$$P(x)=2x^3-5x^2-kx+3,$$

find the value of k.

Find the other two factors of $P(x)$

(c) If the quadratic equation $ax^2+bx+c=0$ has equal roots, solve for x in terms of a and b, where a, b, c $\in R$.

By letting $x=3^y$, write

$$t3^y+3^{-y}=3$$

as a quadratic equation in x, where $t \in R$ and $t \neq 0$.
Find the value of t for which this equation has equal roots.

Assuming this value of t, solve the equation

$$t3^y+3^{-y}=3.$$

Solution

Q1. (a) Solve for x and y:

$$\frac{2x-5}{3}+\frac{y}{5}=6$$

$$\frac{3x}{10}+2=\frac{3y-5}{2}$$

$$\frac{2x-5}{3}+\frac{y}{5}=\frac{6}{1}$$

$$\frac{5(2x-5)+3y=6(15)}{15}$$

$$10x-25+3y=90$$

$$10x+3y=115$$

$$\frac{3x}{10}+\frac{2}{1}=\frac{3y-5}{2}$$

$$\frac{3x+20=5(3y-5)}{10}$$

$$3x+20=15y-25$$

$$3x-15y=-45$$

$$10x+3y=115$$
$$3x-15y=-45$$

Solve the simultaneous equations to find $x=10$ and $y=5$

(b) If $(2x-1)$ is a factor of the polynomial

$$P(x) = 2x^3 - 5x^2 - kx + 3,$$

find the value of k.

Find the other two factors of $P(x)$

If $2x-1$ is a factor then $x = \frac{1}{2}$ is a root.

$$P(x) = 2x^3 - 5x^2 - kx + 3$$

$$P\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 5\left(\frac{1}{2}\right)^2 - k\left(\frac{1}{2}\right) + 3 = 0$$

$$2\left(\frac{1}{8}\right) - 5\left(\frac{1}{4}\right) - \frac{k}{2} + 3 = 0$$

$$\frac{1}{4} - \frac{5}{4} - \frac{k}{2} + 3 = 0$$

$$1 - 5 - 2k + 12 = 0$$

$$-2k = -8$$

$$k = 4$$

$$\begin{array}{rl}
 & x^2 - 2x - 3 \\
2x-1 \overline{\smash{)}} & 2x^3 - 5x^2 - 4x + 3 \\
 & \underline{2x^3 - x^2} \quad \text{change the sign} \\
 & -4x^2 - 4x \\
 & \underline{-4x^2 + 2x} \quad \text{change the sign} \\
 & -6x + 3 \\
 & \underline{-6x + 3} \\
 & 0
\end{array}$$

Factors are $(2x-1)(x^2 - 2x - 3) = (2x-1)(x+1)(x-3)$

(c) If the quadratic equation $ax^2 + bx + c = 0$ has equal roots, solve for x in terms of a and b, where a, b, c $\in R$.

By letting $x = 3^y$, write

$$t3^y + 3^{-y} = 3$$

as a quadratic equation in x, where $t \in R$ and $t \neq 0$.
Find the value of t for which this equation has equal roots.

Assuming this value of t, solve the equation

$$t3^y + 3^{-y} = 3.$$

$ax^2 + bx + c = 0$ has equal roots means $b^2 - 4ac = 0$

Solve $ax^2 + bx + c = 0$ so we use the quadratic formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ but } b^2 - 4ac = 0$$

$$x = \frac{-b}{2a}$$

$$t3^y + 3^{-y} = 3$$

$$t3^y + \frac{1}{3^y} = 3$$

We let $x = 3^y$

$$tx + \frac{1}{x} = 3$$

$$tx^2 + 1 = 3x$$

$$tx^2 - 3x + 1 = 0$$

Equal roots means $b^2 - 4ac = 0$

$$9 - 4t = 0$$

$$t = \frac{9}{4}$$

$$\frac{9}{4}x^2 - 3x + 1 = 0$$

$$9x^2 - 12x + 4 = 0$$

$$(3x-2)(3x-2) = 0$$

$$x = \frac{3}{2}$$

$$3^y = \frac{3}{2}$$

$$\log 3^y = \log \frac{3}{2}$$

$$y \log 3 = \log 3 - \log 2$$

$$y = \frac{\log 3 - \log 2}{\log 3}$$