

6th Year

Test 1

Q1. (a) Show that $\frac{x}{x-4} + \frac{4}{4-x}$ simplifies to a constant where $x \neq 4$

$$\frac{x}{x-4} + \frac{4}{4-x} = \frac{x-4}{x-4} = 1$$

$(x-4)(4-x)$
 $4-x = x-4$

(b)

Solve for x:

$$x = \sqrt{7x-6} + 2.$$

$$\left[\begin{array}{l} x^2 = 7x - 6 + 2 \\ x = \sqrt{7x-6} + 2 \end{array} \right]$$

Learn

$$1 = \sqrt{1} + 2$$

$$(x-2)^2 = x^2 - 4x + 4$$

$$x-2 = \sqrt{7x-6}$$

$$x^2 - 4x + 4 = 7x - 6$$

$$x^2 - 11x + 10 = 0$$

$$\cancel{x=1} \quad x=10$$

Q2. (a)

Solve for x

$$\frac{2x-7}{x+3} < 1, \quad x \neq -3.$$

Learn

$$\frac{2x-7}{x+3} (x+3)^2 < (x+3)^2$$

$$2x^2 + 6x - 7x - 21 < x^2 + 6x + 9$$

$$2x^2 - x - 21 - x^2 - 6x - 9 < 0$$

$$x^2 - 7x - 30 < 0$$

$$x^2 - 7x - 30 = 0$$

$$(x-10)(x+3) = 0$$

$$x = 10 \quad x = -3$$

(b)

(i) Solve for x :

$$|4x+7| < 1.$$

Sq both side

$$16x^2 + 56x + 49 < 1$$

$$16x^2 + 56x + 48 < 0$$

$$4x^2 + 14x + 12 < 0$$

$$2x^2 + 7x + 6 < 0$$

$$2x^2 + 7x + 6 = 0$$

$$(2x+3)(x+2) = 0$$

$$x = -\frac{3}{2} \quad x = -2$$

$$x = -2$$



$$-2 < x < -\frac{3}{2}$$

- (ii) Given that $x^2 - ax - 3$ is a factor of $x^3 - 5x^2 + bx + 9$ where $a, b \in \mathbf{R}$, find the value of a and the value of b .

$$(x+k)(x^2-ax-3) = x^3 - 5x^2 + bx + 9$$

$$x^3 - ax^2 - 3x + kx^2 - akx - 3k = x^3 - 5x^2 + bx + 9$$

$$-a + k = -5 \quad k = a - 5$$

$$-3 - ak = b \quad -3 + 5 = a \quad a = -8$$

$$-3k = 9 \quad a = 2$$

$$k = -3 \quad -3 - 2(-3) = b$$

$$-3 + 6 = b \quad b = 3$$

Q3. (a) Simplify without a calculator

Learn

$$\frac{5+\sqrt{3}}{5-\sqrt{3}} \cdot \frac{5+\sqrt{3}}{5+\sqrt{3}}$$

$$(5+\sqrt{3})(5+\sqrt{3}) = 25 + 5\sqrt{3} + 5\sqrt{3} + 3$$

$$28 + 10\sqrt{3}$$

$$(5-\sqrt{3})(5+\sqrt{3}) = 25 - 3 = 22$$

$$\frac{28 + 10\sqrt{3}}{22}$$

(b)

Solve for

$$3x + y - z = 3$$

$$2x - y + 3z = 20$$

$$7x + y + z = 23$$

$$\begin{array}{r}
 3x + y - z = 3 \\
 2x - y + 3z = 20 \\
 \hline
 5x + 2z = 17
 \end{array}$$

$x = 3$
 $z = 4$
 $y = -2$

Q4. (a)

(a) Solve the simultaneous equations:

$$3x - y = 8$$

$$x^2 + y^2 = 10$$

$$3x - 8 = y$$

$$x^2 + (3x - 8)^2 = 10$$

$$x^2 + 9x^2 - 48x + 64 - 10 = 0$$

$$x = \frac{9}{5}$$

$$x = 3$$

$$y = 3x - 8$$

$$y = 3(3) - 8 = 1$$

$$y = 3\left(\frac{9}{5}\right) - 8 = -\frac{13}{5}$$

(b)

$(x + 1)$ is a factor of $x^3 + 5x^2 + kx - 12$.

Leven

Find the value of k and the other two factors of the cubic expression.

$$x = -1$$

$$(-1)^3 + 5(-1)^2 + k(-1) - 12 = 0$$

$$-1 + 5 - k - 12 = 0$$

$$k = -8$$

$$x+1 \overline{) \begin{array}{r} x^2 + 4x - 12 \\ x^3 + 5x^2 - 8x - 12 \\ \underline{-x^3 + 1x^2} \\ 4x^2 - 8x - 12 \end{array}}$$

$$(x-2)(x+6)$$

$$\begin{array}{r} 4x^2 - 8x \\ \underline{4x^2 + 4x} \\ -12x - 12 \end{array}$$

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