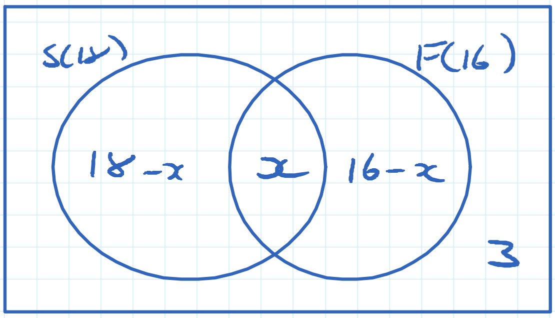
Set and Prob

Class of 30, 18 play soccer, 16 football. 3 no sport. Show on Venn diagram. Find prob play both?

$U(30)$ $S(18)$ $F(16)$ $N(3)$

$\# U = 30$ $\quad$ # = Cardinal number.

$U(30)$



$18 - x + x + 16 - x + 3 = 30$

$x = 7$

$$P(\text{Both}) = \frac{7}{30}$$

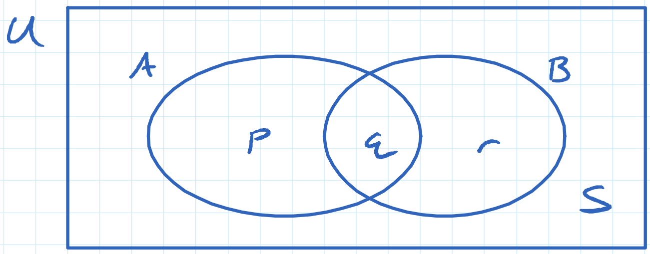
$$P(S) = \frac{18}{30} = \frac{3}{5} \qquad P(F) = \frac{16}{30} = \frac{8}{15}$$

$$\text{Both} = S \text{ AND } F = \frac{3}{5} \times \frac{8}{15} = \frac{8}{25}$$

$$\frac{7}{30} = \frac{8}{25}$$

$$P(S \cap F) \neq P(S) \cdot P(F)$$

$\Rightarrow$ S and F are not independent.



$P(U) = 1 = p+q+r+s$

$P(A\cup B) = P(A)+P(B)-P(A\cap B)$

$= p+q+r$

$P(A) =$ in $A = p+q$

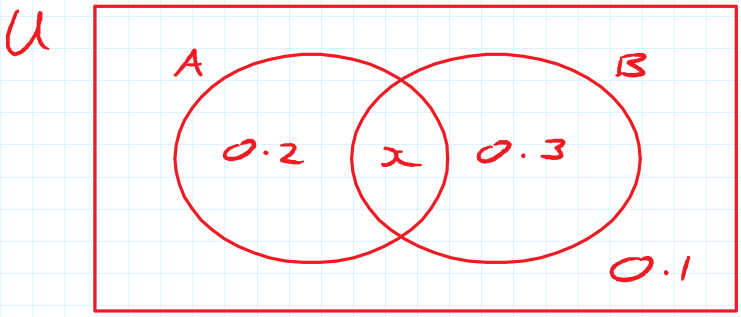
$P(A') =$ outside $= r+s$

$P(A|B) = A$ given B has occured

$$= \frac{P(A\cap B)}{P(B)} = \frac{q}{q+r}$$

Mutually exclusive $\Rightarrow$ nothing in common $\Rightarrow P(A\cap B) = 0$

Exhaustive $\Rightarrow P(A)+P(B) = 1$



Find x.

State

(i) $P(A)$

(ii) $P(A\cap B)$

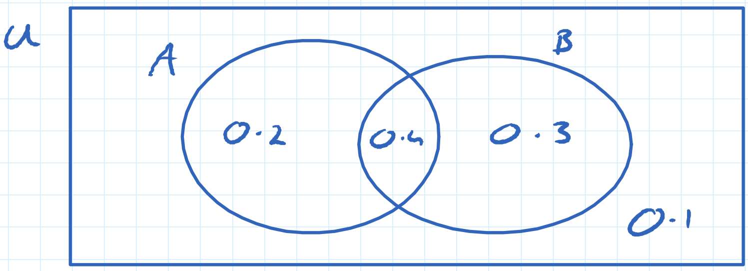
(iii) $P(A')$

(iv) $P(A\cup B)$

(v) Are A and B independent.

$0.2 + x + 0.3 + 0.1 = 1$

$x = 0.4$



(i) $P(A) = 0.6$ $P(B) = 0.7$

(ii) $P(A\cap B) = 0.4$

(iii) $P(A') = 0.4$

(iv) $P(A\cup B) = 0.$

$0.6 + 0.7 - 0.4 = 0.$

(v) $P(A) = 0.6$

$P(A)\times P(B) = 0.42 \neq 0.4 = P(A\cap B)$

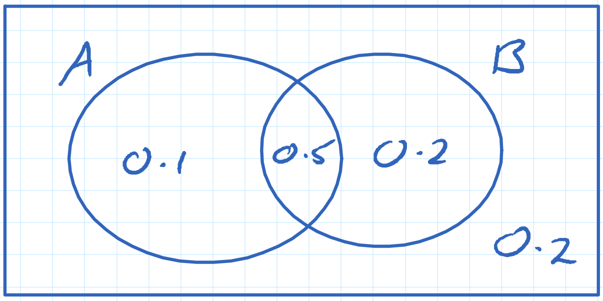
not independent.

$P(A) = 0.6$, $P(B) = 0.7$, $P(A \cap B) = 0.5$

Find $P(A \cup B)$. Are A and B

(i) independent?

(ii) mutually exclusive?



$P(A \cup B) = 0.1 + 0.5 + 0.2$

$= 0.8$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.6 + 0.7 - 0.5$

$= 0.8$

(i) Independent

Method 1: $P(A \cap B) = P(A) \times P(B)$

$0.5 = 0.6 \times 0.7$

$0.5 = 0.42$

$\Rightarrow$ not independent

Method 2:

If independent $P(A) = P(A|B)$

$P(A) = 0.6$

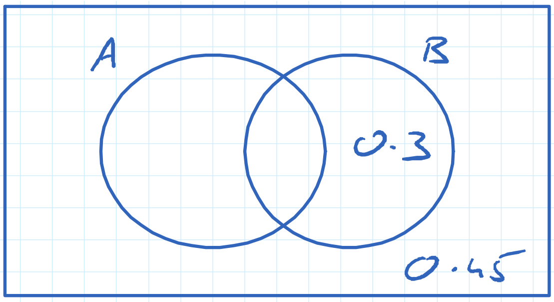
$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.5}{0.7} = 0.7$$

$$P(A) \neq P(A|B)$$

$$\Rightarrow \text{not independent.}$$

A and B are independent events such that $P(A) = 0{\cdot}25$ and $P(A \cup B) = 0{\cdot}55$.

Find $P(B)$.



$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$0.55 = 0.25 +$$

$$P(A\cap B) = P(A)\,P(B)$$

$$\cancel{= 0.25(0.3)}$$

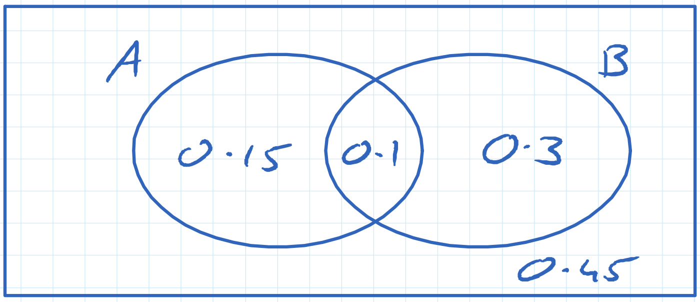
$$\cancel{= 0.075}$$

$$P(B) = x$$

$$0.55 = 0.25 + x - 0.25x$$

$$0.3 = 0.75x$$

$$x = 0.4$$



A bag contains the following cardboard shapes:
10 red squares, 15 green squares, 8 red triangles and 12 green triangles.
One of the shapes is drawn at random from the bag.
E is the event that a square is drawn.
F is the event that a green shape is drawn.

(i) Find $P(E \cap F)$.

(ii) Find $P(E \cup F)$.

(iii) State whether E and F are independent events, giving a reason for your answer.

(iv) State whether E and F are mutually exclusive events, giving a reason for your answer.

10 RS 15 GS 8 RT 12 GT Total 45

(i) $P(E \cap F)$ = Square and Green $= \frac{15}{45}$

(ii) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= \frac{25}{45} + \frac{27}{45} - \frac{15}{45} = \frac{37}{45}$

(iii) $P(E \cap F) = P(E) \times P(F)$

$\frac{15}{45} = \frac{25}{45} \cdot \frac{27}{45}$ True

$\Rightarrow$ independent

(iv) No $P(E \cap F) \neq 0$

Two events E_1 and E_2 are independent. $P(E_1) = \frac{1}{5}$ and $P(E_2) = \frac{1}{7}$. Find

(i) $P(E_1 \cap E_2)$

(ii) $P(E_1 \cup E_2)$.

(i) $P(E_1 \cap E_2) = P(E_1) \times P(E_2)$

$= \frac{1}{5} \times \frac{1}{7} = \frac{1}{35}$

(ii) $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{7}{35} + \frac{5}{35} - \frac{1}{35} = \frac{11}{35}$