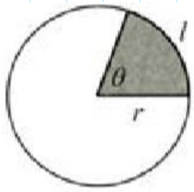
Circle

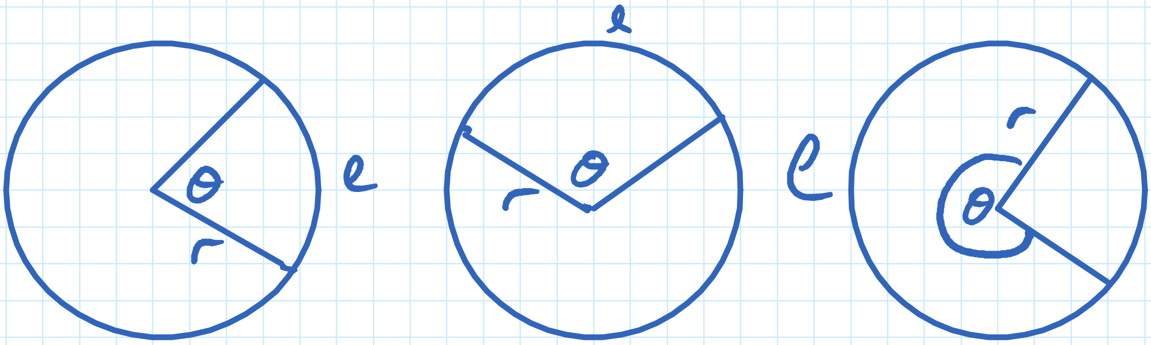
Tables.

Stua / Teascóg **Arc / Sector**

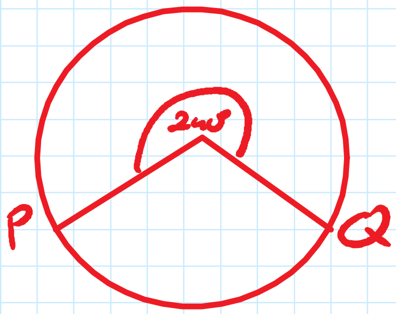


nuair is ina raidiain atá θ	$l = r\theta$	$A = \frac{1}{2}r^2\theta$	*when θ is in radians*
nuair is ina chéimeanna atá θ	$l = 2\pi r\left(\frac{\theta}{360^\circ}\right)$	$A = \pi r^2\left(\frac{\theta}{360^\circ}\right)$	*when θ is in degrees*

l = arc length.



$l = r\theta$ $l = r\theta$ $l = r\theta$



r = 10 cm.
Find major arc PQ to 1 decimal place.

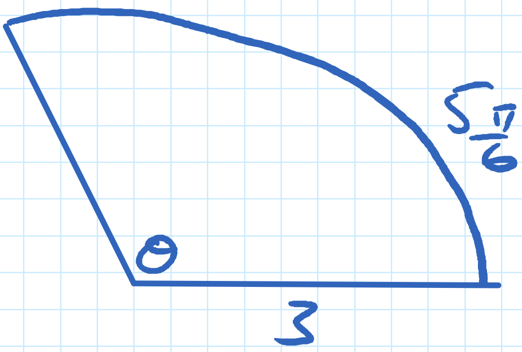
$A = 2\pi r \left(\frac{\theta}{360}\right)$

$= \frac{240}{360}(2\pi)(10)$

$= 41.9$ cm

Use π on calculator unless told otherwise.

A sector has a radius of 3 cm and length of $\frac{5\pi}{6}$ find area to 1 decimal place.



$A = \frac{1}{2}r^2\theta$

$l = \frac{5\pi}{6}$

$r\theta = l$

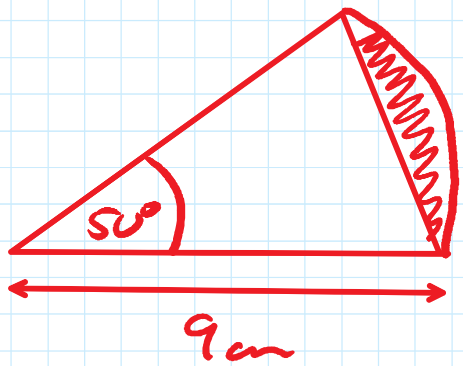
$3\theta = \frac{5\pi}{6}$

$\theta = \frac{5\pi}{18}$

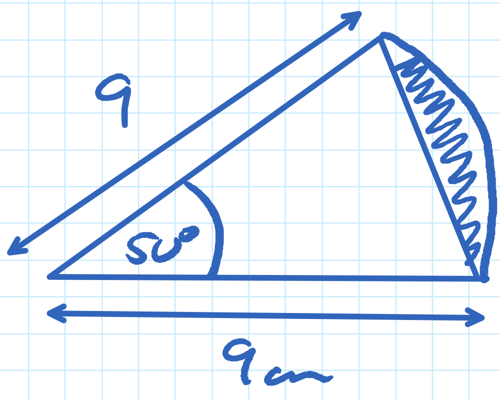
$\text{Area} = \frac{1}{2}(3)^2\left(\frac{5\pi}{18}\right)$

$= \frac{5\pi}{4}\ \text{cm}^2$

$= 3.9\ \text{cm}^2$



Find shaded area to 1 decimal place.

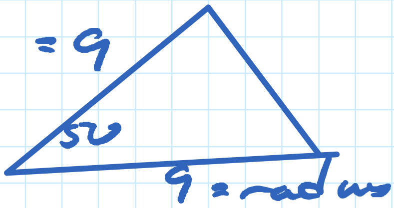


Area of sector − Area of triangle.

$\text{Sector} = \frac{\theta}{360}\pi r^2$

$= \frac{50}{360}\pi(9)^2 = 35.3$

Radius = 9



Area = $\frac{1}{2}ab\sin c$

$= \frac{1}{2}(81)\sin 50$

$= 31.0$

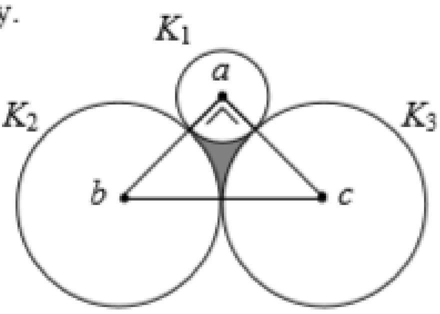
Answer $= 35.3 - 31$

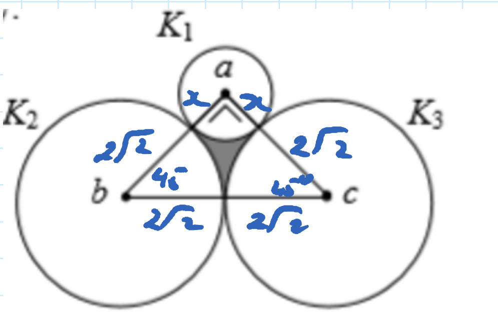
$= 4.3\text{ cm}^2$

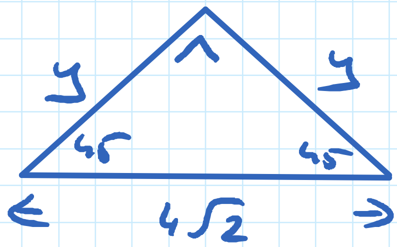
a, b and c are the centres of circles K_1, K_2 and K_3 respectively.
The three circles touch externally and $ab \perp ac$.
K_2 and K_3 each have radius $2\sqrt{2}$ cm.

(i) Find, in surd form, the length of the radius of K_1.

(ii) Find the area of the shaded region in terms of π.







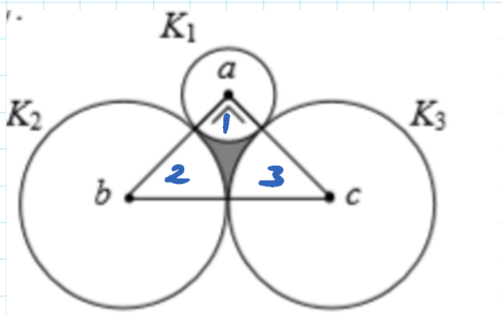
$y^2 + y^2 = 16(2)$

$2y^2 = 16(2)$

$y^2 = 16$

$y = 4$

$x = 4 - 2\sqrt{2}$



Triangle − 3 sectors.

Area = $\frac{1}{2}$ base × height

$$= \frac{1}{2}(4)(4) = 8\ cm^2$$

$$A = \left(\frac{\theta}{360}\right)\pi r^2$$

$$= \frac{1}{4}\pi(4-2\sqrt{2})^2$$

$$= (6-4\sqrt{2})\pi$$

$$A = \frac{1}{8}(2\sqrt{2})^2\pi = \pi$$

3 sectors $= (6-4\sqrt{2})\pi + 2\pi$

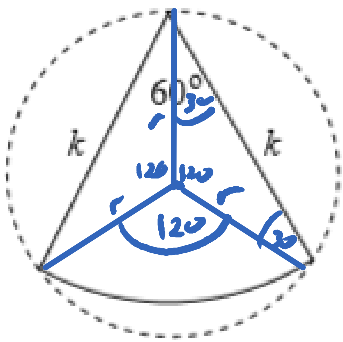
$$(8-4\sqrt{2})\pi$$

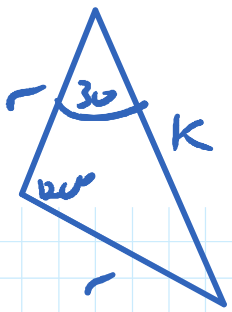
Shaded $= 8 - (8-4\sqrt{2})\pi\ .\ cm^2$

The diagram shows a sector (solid line) circumscribed by a circle (dashed line).

(i) Find the radius of the circle in terms of k.

(ii) Show that the circle encloses an area which is double that of the sector.





$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{r}{\sin 30} = \frac{k}{\sin 120}$$

$$r = \frac{k \sin 30}{\sin 120}$$

$$r = \frac{\sqrt{3}k}{3}$$

Circle $A = \pi r^2 = \pi\left(\frac{\sqrt{3}k}{3}\right)^2$ $\left(\frac{\sqrt{3}}{3}\right)^2 = \frac{3}{9}$

$= \frac{\pi k^2}{3}$

Sector $A = \frac{60}{360}\pi k^2 = \frac{\pi k^2}{6}$

Circle = 2 sector.

$\frac{\pi k^2}{3} = \frac{2\pi k^2}{6}$

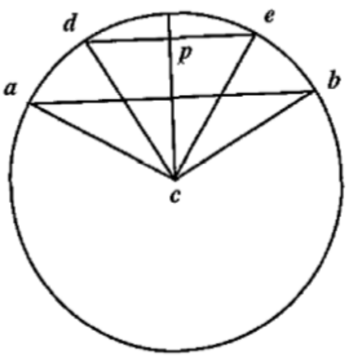
[ab] and [de] are two parallel chords of a circle with centre c and radius length r.
$cp \perp de$, $|\angle acb| = 4\beta$ and $|\angle dce| = 2\beta$, where β is in radian measure, $\beta \neq 0$.

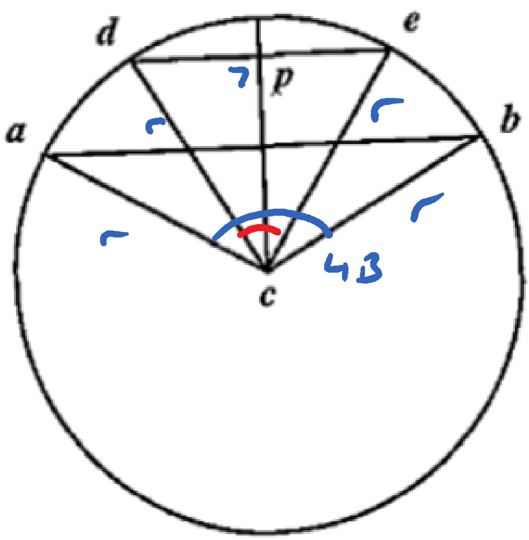
(i) If the area of the triangle acb = the area of triangle dce, show that $\beta = \frac{\pi}{6}$.

(ii) Calculate the value of r if

$|ab|^2 + |de|^2 = 24$

and give your answer as a surd.





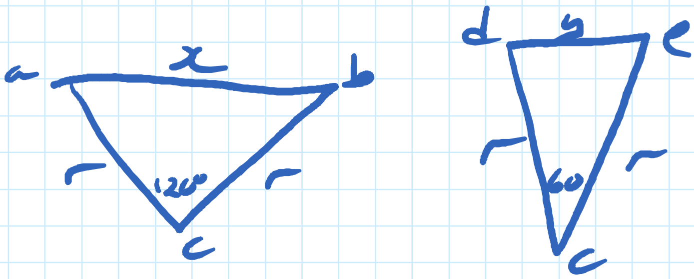
Area = $\frac{1}{2}ab\,\mathrm{Sin}\,C$

$\frac{1}{2}r^2\,\mathrm{Sin}\,4B = \frac{1}{2}r^2\,\mathrm{Sin}\,2B$

$\mathrm{Sin}\,4B = \mathrm{Sin}\,2B$

$B = \frac{\pi}{6}$ $\mathrm{Sin}\,\frac{4\pi}{6} = \mathrm{Sin}\,\frac{2\pi}{6}$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$



$x^2+y^2=24$

$\cos 120 = -\frac{1}{2}$

$x^2 = r^2+r^2-2r^2\cos 120 = 3r^2$

$y^2 = r^2+r^2-2r^2\cos 60 = r^2$

$4r^2 = 24$

$r^2 = 6$

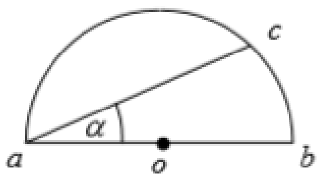
$r = \sqrt{6}$

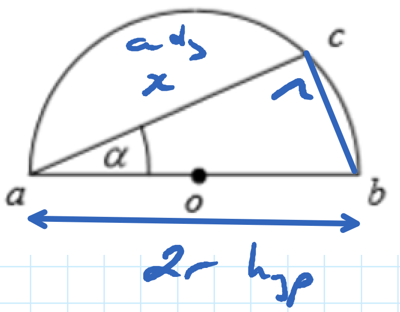
$[ab]$ is the diameter of a semicircle of centre o and radius-length r.
$[ac]$ is a chord such that $|\angle cab| = \alpha$, where α is in radian measure.

(i) Find $|ac|$ in terms of r and α.

(ii) $[ac]$ bisects the area of the semicircular region.

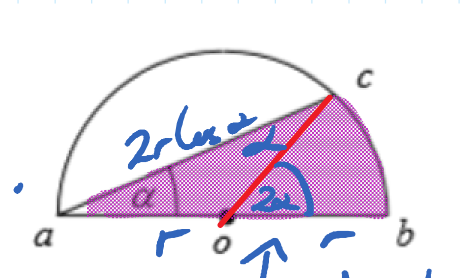
Show that $2\alpha + \sin 2\alpha = \frac{\pi}{2}$.





$\cos\alpha = \frac{x}{2r}$

$2r\cos\alpha = x$



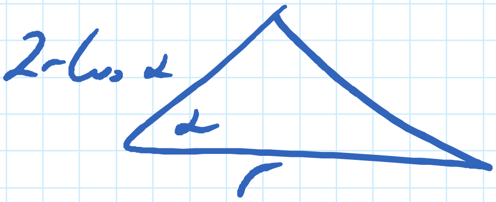
Semi-circle

$$A = \frac{1}{2}\pi r^2$$

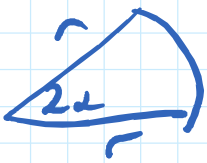
Exterior = sum of interior.

Shaded area $= \frac{1}{4}\pi r^2$

Triangle + Sector of a circle.



$$\text{Area} = \frac{1}{2} r (2r\cos\alpha)\sin\alpha$$
$$= r^2 \cos\alpha \sin\alpha$$



$$\text{Area} = \frac{1}{2} r^2 \theta$$
$$= \frac{1}{2} r^2 (2\alpha) = r^2\alpha$$

$$r^2\cos\alpha\sin\alpha + r^2\alpha = \frac{\pi r^2}{4}$$

$$\cos\alpha\sin\alpha + \alpha = \frac{\pi}{4}$$

$$2\cos\alpha\sin\alpha + 2\alpha = \frac{\pi}{2}$$

$$\sin 2\alpha + 2\alpha = \frac{\pi}{2}.$$