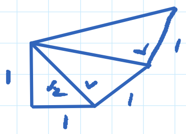
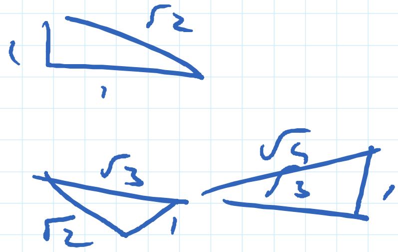
Surds = Square Roots.

$\sqrt{a}$ = irrational number

= $\mathbb{R} | \mathbb{Q}$ = real less rational.

Construct $\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}$ .....





Rules

$$\sqrt{a^2} = (\sqrt{a})^2 = a$$

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b} \qquad \sqrt{9+16} \neq \sqrt{9} + \sqrt{16}$$

$$\sqrt{25} \neq 3+4$$

$$5 \neq 7$$

Simple surds $a\sqrt{b}$

a = rational

b = irrational.

Compound (Complex) Surds

$$a + \sqrt{b}$$

$$a + b\sqrt{c}$$

Add

Simplify (i) $2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$

(ii) $5\sqrt{2} - 9\sqrt{2} = -4\sqrt{2}$

(iii) $\sqrt{12} + \sqrt{27}$ $\qquad \sqrt{ab} = \sqrt{a}\sqrt{b}$

Multiply. Simplify

$$(2+\sqrt{3})(5+\sqrt{3})$$

$$10 + 2\sqrt{3} + 5\sqrt{3} + 3$$

$$13 + 7\sqrt{3}$$

Simplify

$$(2+\sqrt{3})(2-\sqrt{3})$$

$$4 - \cancel{2\sqrt{3}} + \cancel{2\sqrt{3}} - 3 = 1$$

$$(a+b)(a-b) = a^2 - b^2$$

Every compound surd has a surd conjugate :- change sign in middle.

Compound Surd	Conjugate
$a + \sqrt{b}$	$a - \sqrt{b}$
$\sqrt{p} - \sqrt{q}$	$\sqrt{p} + \sqrt{q}$

Division

Simplify $\frac{1}{\sqrt{3}}$

~~$\frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} = \frac{1}{3}$~~ change

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

(ii) $\frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

This is called rationalising the denominator.

(iii) $\frac{1}{3+\sqrt{2}}$

~~$\frac{1}{3+\sqrt{2}} \quad \frac{3+\sqrt{2}}{3\sqrt{2}}$~~

$\sqrt{a}\sqrt{b} = \sqrt{ab}$

$\frac{1}{3+\sqrt{2}} \cdot \cancel{\frac{\sqrt{2}}{\sqrt{2}}}$

~~$\sqrt{2}\sqrt{1} = \sqrt{2}$~~

~~$\frac{1}{3+\sqrt{2}} \cdot \frac{3+\sqrt{2}}{3+\sqrt{2}}$~~

$(3+\sqrt{2})(3+\sqrt{2}) = 9 + 3\sqrt{2} + 3\sqrt{2} + 2$

$$\frac{1}{3+\sqrt{2}} \quad \frac{3-\sqrt{2}}{3-\sqrt{2}}$$

$$\frac{3-\sqrt{2}}{7}$$

$(3+\sqrt{2})(3-\sqrt{2})$

$3^2 - (\sqrt{2})^2$

$9 - 2 = 7$

Simplify

(i) $\dfrac{3+\sqrt{5}}{3-\sqrt{5}}$

$$\frac{3+\sqrt{5}}{3-\sqrt{5}}$$

$$\frac{3+\sqrt{5}}{3-\sqrt{5}} \cdot \frac{3+\sqrt{5}}{3+\sqrt{5}}$$

Top $(3+\sqrt{5})(3+\sqrt{5})$

$$9 + 3\sqrt{5} + 3\sqrt{5} + 5$$

$$14 + 6\sqrt{5}$$

Bottom $(3-\sqrt{5})(3+\sqrt{5}) = 9 - 5 = 4$

$(a-b)(a+b) = a^2 - b^2$

$$\frac{14+6\sqrt{5}}{4} = \frac{7+3\sqrt{5}}{2}$$

$$= \frac{7}{2} + \frac{3}{2}\sqrt{5}$$

(ii) $\dfrac{5+\sqrt{2}}{5-\sqrt{2}}$

$$\frac{5+\sqrt{2}}{5-\sqrt{2}} \cdot \frac{5+\sqrt{2}}{5+\sqrt{2}}$$

$$25 + 5\sqrt{2} + 5\sqrt{2} + 2 = 27 + 10\sqrt{2}$$

$$(5-\sqrt{2})(5+\sqrt{2}) = 25 - 2 = 23$$

$$\frac{27+10\sqrt{2}}{23}$$

(iii) $\dfrac{27}{\sqrt{18}}$

$\sqrt{18} = \sqrt{9}\sqrt{2}$

$3\sqrt{2}$

$\dfrac{\cancel{27}}{\cancel{3}\sqrt{2}} = \dfrac{9}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{9\sqrt{2}}{2}$